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AN APPLICATION OF TIME-OPTIMAL
CONTROL THEORY TO LAUNCH VEHICLE REGULATION

By

F. B. Smith, Jr., and J. A. Lovingood

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SUMMARY

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A minimum response time criterion is used in the design of a pitch-attitude controller for a flexible launch vehicle. The criterion is applied to a fourth-order model containing the primary dynamics of the thirteenth-order vehicle. A collection of open-loop optimal trajectories is used to define a closed-loop control law. Results of an analog simulation are presented which show that this control law properly applied to the flexible vehicle results in good control.

In spite of the relatively large effort which has gone into the study of optimization during the last few years, there have been few applications to significant, practical closed-loop control problems. This is true in spite of the fact that theoretical developments promise solutions or potential solutions to control problems for which conventional synthesis procedures are not completely satisfactory. Among the difficulties which have hindered practical applications are: adequate description of real plants often requires differential equations of quite high order, the control law is usually a nonlinear function of many variables and difficult to implement, and the theoretical solution of the optimization problem most often yields the open-loop law, $u(t, x(0))$, rather than the required closed-loop law, $u(x)$. The flexible launch vehicle is used in this paper to illustrate these problems and to demonstrate the use of some techniques to overcome them.

EQUATIONS OF MOTION

The assumed equations of a typical 250,000-lb flexible launch vehicle are given in Table 1. Poles and zeroes of the θ_R/u transfer function are listed in Table 2. Airframe coefficients are taken at the maximum dynamic pressure flight condition with flight speed assumed constant. The equations include dynamics of the rigid body, three-body flexure modes, tail-wags dog, actuator, and rate servo and an integration of pitch rate for control of pitch attitude. A single control variable is assumed available from gimbaling of the engine. A maximum gimbal rate of 0.2 rad/sec is commanded at all times.

SPECIFICATION OF THE CONTROLLER

In applying optimal control theory to the synthesis of controllers for practical plants, it is necessary to specify both the optimization criterion and what is to be controlled. With the criterion of minimum response time used here it has been common to apply the criterion to the state vector y of a plant in the form

$$\dot{y} = Ay + Bu. \quad (1)$$

However, when this is done, the resulting response in multidegree of freedom systems may be entirely unacceptable. This is forcefully demonstrated by time-optimally regulating the state vector of the rigid launch vehicle given in Figure 1. When all components of the state vector, pitch attitude, pitch rate, angle of attack, and gimbal deflection are brought to zero in minimum time from an initial displacement in pitch attitude of 0.01 rad displacements of attitude and angle of attack greater than 0.15 rad occur. Although this is the time-optimal response for regulation of the state vector, it is certainly not acceptable since it would literally destroy the vehicle.

On the other hand, if the problem posed is that of bringing the single component, pitch attitude, to zero in minimum time and holding it there, the deadbeat response to step input of attitude is obtained (Fig. 1). In this case,

Table 2. Critical Frequencies of $G(s) = \frac{\theta_R(s)}{U(s)}$

Static gain of airframe = 0.0985 rad/rad		
Source	Poles	Zeros
Pseudo - Integrator	-0.02	--
Actuator	-30 -6.5 ± j 65 $\sqrt{0.99}$	--
Rigid Body	1.4296 -1.4964	-0.0478
Tail Wags Dog	--	-0.0233 ± j 57.00
First Bending Mode	-0.0962 ± j 18.00	-0.120 ± j 17.77
Second Bending Mode	-0.233 ± j 46.34	-0.225 ± j 46.81
Third Bending Mode	-0.479 ± j 92.69	-0.454 ± j 94.35

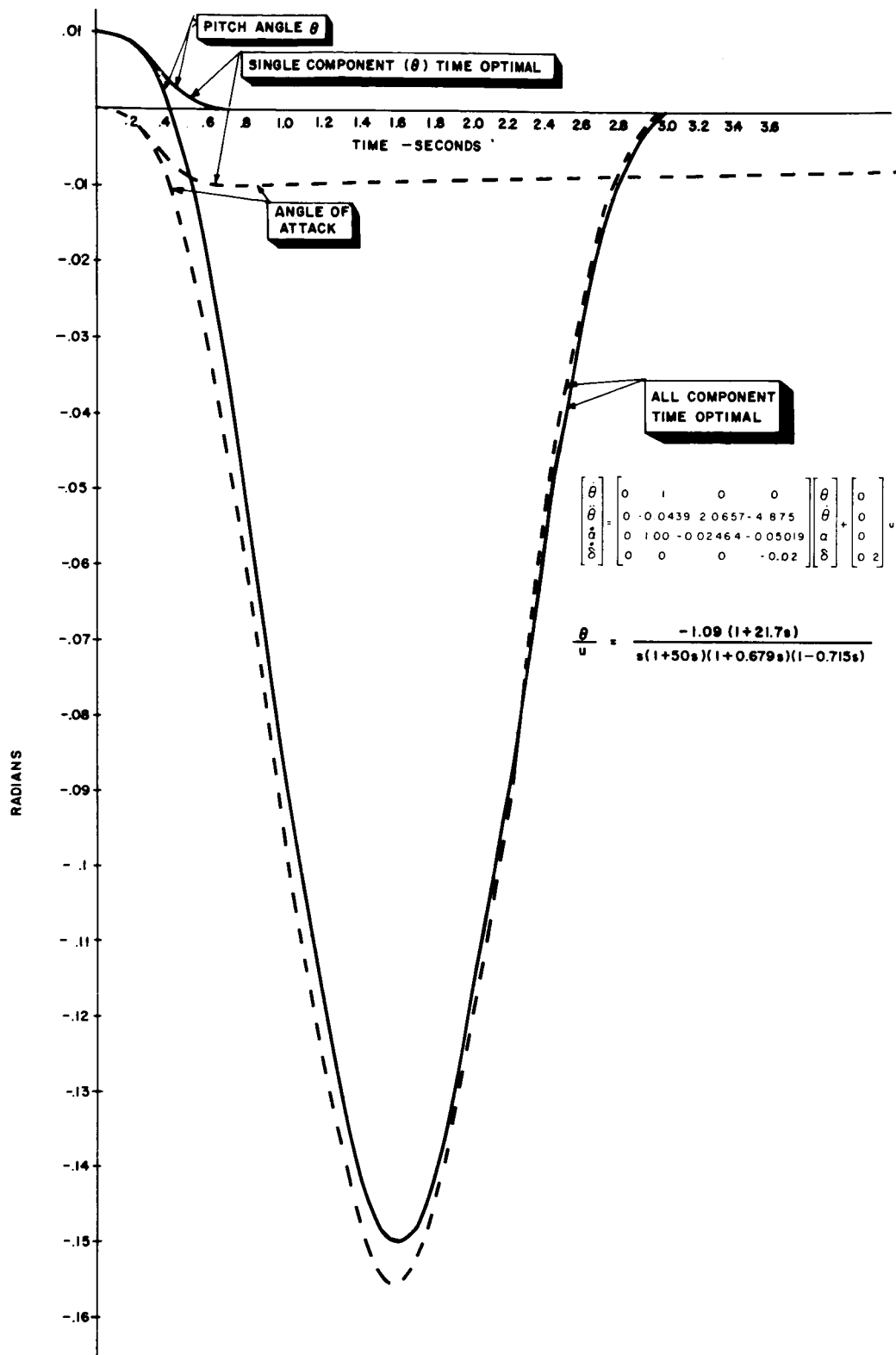


Figure 1. Time Optimal Control of a Rigid Launch Vehicle

angle of attack and gimbal deflection are not zero at the response time (time when θ and $\dot{\theta}$ are first zero), but decay with a 21.7-sec time constant characteristic of the plant. It has been shown that single-component control can be described as motion to a region in the n-dimensional space. The target region is determined as that region in n-space where the component being controlled is zero and is capable of being held there with a bounded control variable (Ref 1 and 2). The necessary and sufficient conditions for minimum time motion to such a region have been obtained (Ref 3).

In the work presented in this paper, optimum control synthesis techniques are demonstrated for control of pitch attitude. The controller obtained is fourth order, one dimensional. That is, the control variable is a function of four variables, and the target set is a line segment in this four-space. Choice of pitch attitude was arbitrary. The techniques apply equally well to control of other components of the state vector or to control of a linear combination of them such as minimum drift.

A TRUNCATED MODEL

Although time-optimal control theory applies in principle to regulation of plants of any order, it is not desirable nor necessary to apply it in controller design to the complete plant representation when the motion of the variable being controlled is primarily influenced by relatively few variables. In the launch vehicle considered, the flexure mode frequencies are quite high and aerodynamic coupling is small, so flexure has only minor effects on rigid-body pitching motion. The same is true of the actuator dynamics. Consequently, there is a natural division of the plant into a set of dominant and a set of secondary dynamics. Time-optimal synthesis is applied to control the dominant modes only, and conceptually the secondary dynamics act as a filter on the primary modes. This is shown in Figure 2.

The transfer function θ_R/u , for the entire plant of Table 1, has been divided into two parts

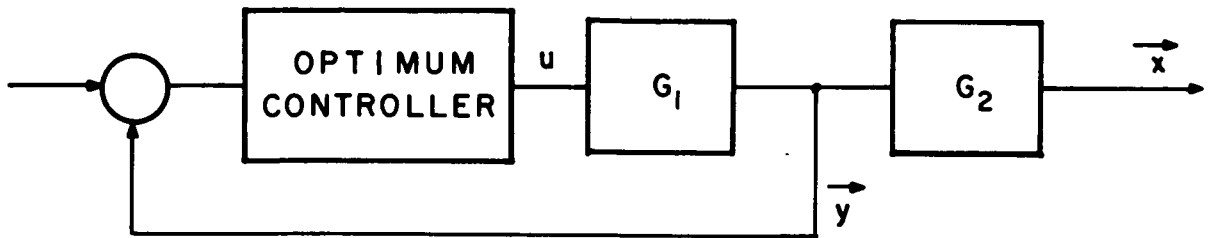


Figure 2. Conceptual Feedback for Controller Synthesis

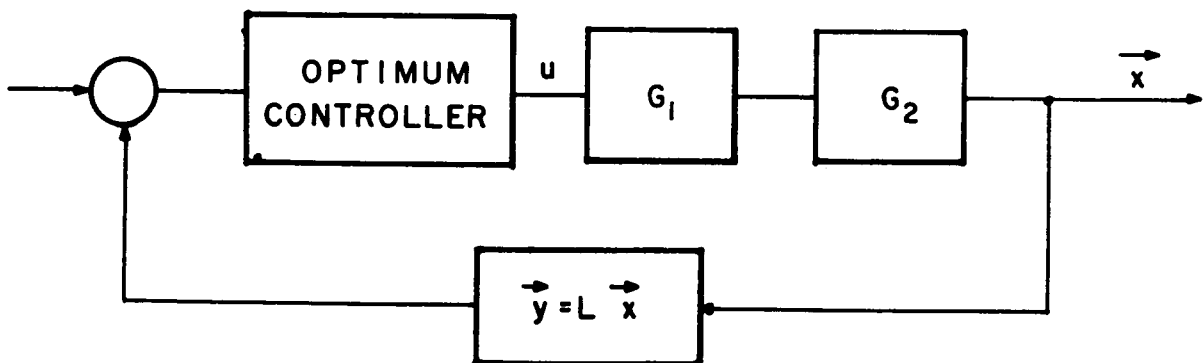


Figure 3. Actual Feedback for Controller Mechanization

$$\frac{\theta_R(s)}{u(s)} = G_1 \cdot G_2. \quad (2)$$

Primary dynamics are contained in

$$G_1 = \frac{0.8808 (s + 0.0478)}{s(s + 0.02) (s - 1.4296) (s + 1.4964)} \quad (3)$$

and secondary dynamics in G_2 . Feedback of the fictitious output of G_1 is used for controller design. The partial principle coordinate methods of Reference 4 permit deriving the linear transformation relating the y-coordinates to the state of the system, x . The transformation

$$y = Lx \quad (4)$$

where y is an m-vector, L , an mxm-matrix, and x , an n-vector, in general, then permits the fictitious control loop of Figure 2 to be changed to the one which is physically realizable in Figure 3.

A plant in state vector form which gives the transfer function of Eq (3) is

$$\begin{bmatrix} \dot{y}_1 \\ \dot{y}_2 \\ \dot{y}_3 \\ \dot{y}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & -0.0394 & 2.1403 & -4.404 \\ 0 & 1.00 & -0.02738 & -0.04213 \\ 0 & 0 & 0 & -0.02 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ .2 \end{bmatrix} u. \quad (5)$$

This was obtained by deriving $(y_1(s))/(u(s))$ and $(y_3(s))/(u(s))$ transfer functions from a set of equations of this form but with unknown coefficients and then adjusting coefficients to give the proper poles, zeroes, and gains. A similar set of equations could be obtained directly from the transfer functions of Eq (3) and the transformation to continuous coordinates of Reference 5.

The transformation matrix L , which relates the output of the flexible vehicle to the y-variables contains many elements which are small. It is possible to neglect these. The transformation used in the analog simulation was

$$\begin{bmatrix} \theta_F \\ \dot{\theta}_F \\ \alpha_F \\ \delta_F \end{bmatrix} = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix} = \begin{bmatrix} 1 & 0.0341 & 0 & 0 & 0 \\ 0 & 0.999 & 0.0729 & 0 & -0.15 \\ 0 & 0.0341 & 0.999 & 0 & -0.0016 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \theta_R \\ \dot{\theta}_R \\ \alpha_R \\ \delta_c \\ \delta_e \end{bmatrix} \quad (6)$$

It is seen that y_1 corresponds very closely with θ_R , y_2 with $\dot{\theta}_R$, y_3 with α_R and y_4 with δ_c . Motion of θ_F corresponds very closely with that of θ_R , so it is reasonable to take Eq (5) as the truncated model of the full system.

Two points should be emphasized in the choosing of a truncated model for controller design. First, division of the plant into primary and secondary dynamics cannot be made until the variable to be controlled has been specified. This variable may be one of the physical variables appearing in the state vector x or may be a linear combination of them. Second, even if the secondary dynamics are a result of a limited number of physical variables in the equations of motion, Eq (1), the primary dynamics cannot be obtained by simply neglecting these variables and equations. For example, if the equations for the flexure modes and actuator were omitted in truncating to a fourth-order model corresponding to G_1 , the poles at -1.4962 and 1.4296 would be at -1.47 and 1.403.

CLOSED-LOOP TIME-OPTIMAL CONTROL LAW

The next step in the synthesis procedure is to derive a closed-loop controller for the model of Eq (5). The criterion for design is time-optimal regulation of pitch attitude; that is, y_1 is to be brought to zero from an initial condition in minimum time, subject to a bounded control variable, and then held at zero. This corresponds to motion to a one-dimensional line segment in the four-dimensional space of θ_F , $\dot{\theta}_F$, α_F , and δ_F .

There is no known method for obtaining a useful closed-form expression for the closed-loop control law $u(x)$ which moves the plant to the desired line segment optimally. However, it is possible to compute open-loop solutions $u(t, x(0))$ for any initial condition $x(0)$ by the computational

techniques described in Reference 6. These techniques solve a set of transcendental equations for a control variable $u(t, x(0))$ which is constrained to satisfy the maximum principle. Since the maximum principle has been shown to be a necessary and sufficient condition for the optimum solution, the $u(t, x(0))$ obtained is the optimal one. It is not practical to solve the required equations on line to achieve effective closed-loop control. Instead, a collection of open-loop optimum trajectories from a set of initial conditions distributed evenly throughout the phase space region of interest is used to define a closed-loop control law by the method described in Reference 7. Each of the variables θ_F , $\dot{\theta}_F$, α_F , and δ_F is divided up into 32 regions called quanta. A Boolean variable X_i^j , is defined for each quantum ($i = 1, 2, 3, 4$; $j = 1, 2, \dots, 32$). The variable X_i^j takes a value one if the measured magnitude of the i^{th} variable is within the j^{th} region and takes the value zero if the magnitude is within any other region. A logic form,

$$u(x) = \text{sign} \left[\sum_{i=1}^4 \sum_{j=1}^{32} X_i^j \lambda_i^j \right] \quad (7)$$

is assumed capable of mechanizing the control law, and the 128 constants, λ_i^j , are experimentally adjusted to make $u(x)$ agree with the optimum control at discrete points on the optimum trajectory.

This adjustment or training procedure is shown in Figure 4. Switch S is opened at $t = 0$, and the open-loop optimal solution $u(t)$ is applied to the simulated plant. Output of the plant $x(t)$ is the input to the logical net, and the output of the net $u(x(t))$ is compared with the optimum control variable $u(t, x(0))$ at discrete intervals of time. If the control variables are different, λ_i^j corresponding to the X_i^j 's which are one for that $x(t)$ are incremented in the direction to make the sign of their sum the same as the sign of $u(t, x(0))$. If $u(x(t))$ and $u(t, x(0))$ are the same, no adjustment is made. This procedure was carried out on a general purpose digital computer with a set of 198 optimum trajectories for the plant of Eq (5), distributed in the space,

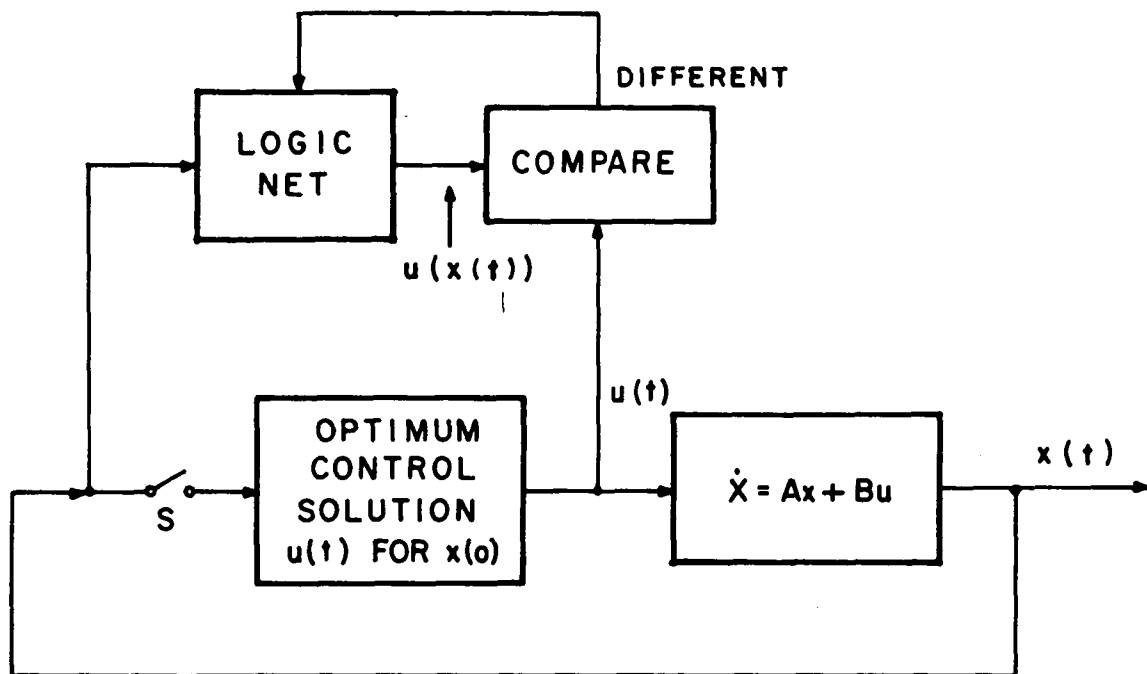


Figure 4. Logic Adjustment Procedure

$$\begin{array}{rclcl}
0 & \leq & \theta_F & \leq & 0.1 \\
-0.12 & \leq & \dot{\theta}_F & \leq & 0.12 \\
-0.1 & \leq & \alpha_F & \leq & 0.1 \\
-0.12 & \leq & \delta_F & \leq & 0.12
\end{array} \tag{8}$$

Control variable comparison points were at intervals of 0.1 sec.

As the adjustment is carried out, the number of differences (called errors) between $u(x(t))$ and $u(t, x(0))$ is an indication of the convergence of the procedure. The percentage of errors, $100 (\text{No. of errors in } N \text{ points}/N)$, is plotted as a function of the number of trajectories in Figure 5. First switch points are those between $t = 0$ and the first switch time, second switch points between the first switch time and the second, etc. Initial λ_i^j 's were all taken to be zero. It is seen that errors drop very rapidly at first, being less than 10% after only 100 trajectories. At 5000 and 7500 trajectories the resolution of the logic of Eq (7) is artificially increased by multiplying all λ_i^j 's by 2. At 11,000 trajectories, the λ_i^j 's are multiplied by a factor of 10. Typical closed-loop control responses by the logic at the stages of training shown in Figure 5 are presented in Figure 6. At 198 trajectories the controller has not yet stabilized the statically unstable vehicle. At 2100 trajectories the closed loop is apparently stable, but responses are poor. At 11,000 trajectories responses closely approximate optimum. (Limited hardware did not permit evaluation of closed-loop responses at 13,500 trajectories.) The logic of Eq (7) with constants at 11,000 trajectories is taken as the closed-loop controller for the plant of Table 1.

CONTROL OF THE FLEXIBLE VEHICLE

A block diagram of the control system is given in Figure 7. Mechanization of the logical net for this optimal control of the fourth-order plant was accomplished by using standard, commercial analog to digital converters for quantization and diode-transistor logic in conjunction with standard ladder networks to form the logic of Eq (7) (Ref 7). A linear switching mode of the control variable was used when the plant output was within approximately 1 quantum of

FOURTH-ORDER, ONE-DIMENSIONAL
TIME-OPTIMAL CONTROL OF THE
PLANT

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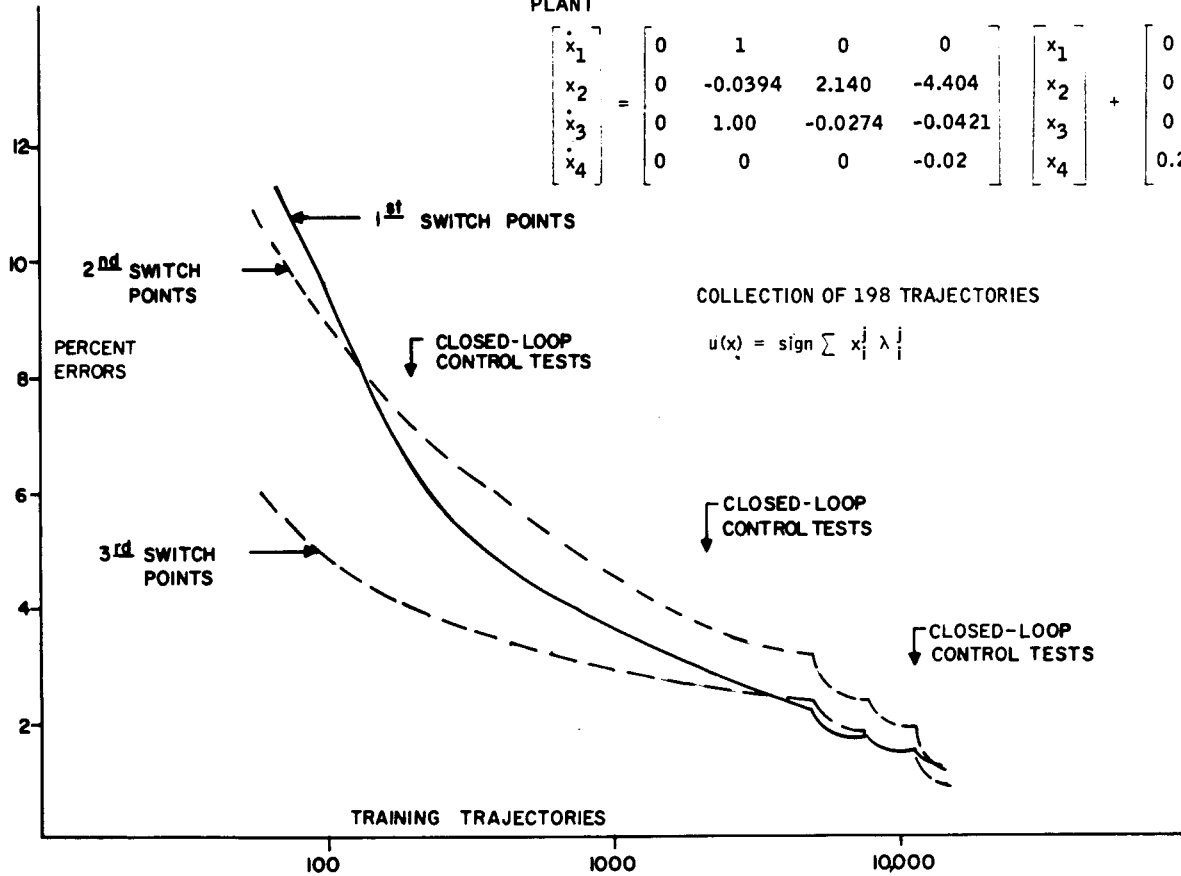


Figure 5. Fourth-order Training Curve

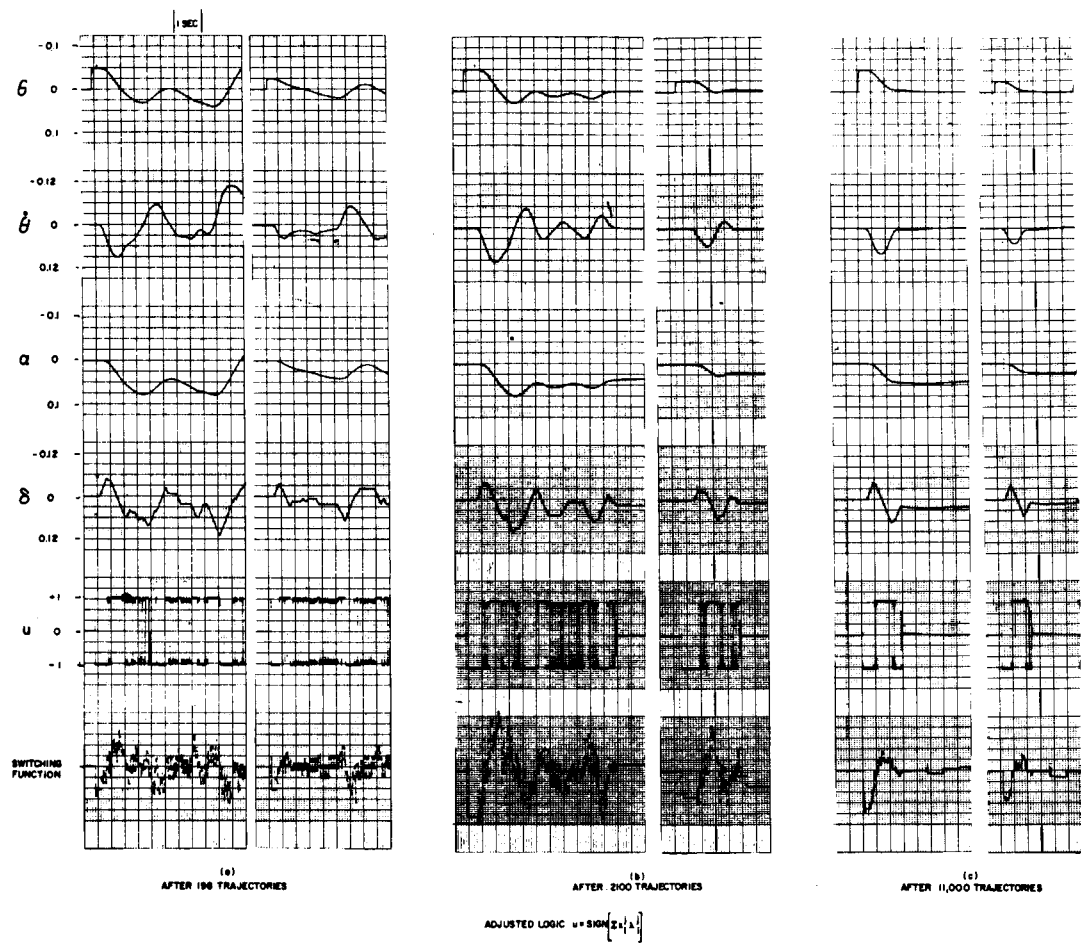


Figure 6. Closed-loop Responses at Three Stages of Training

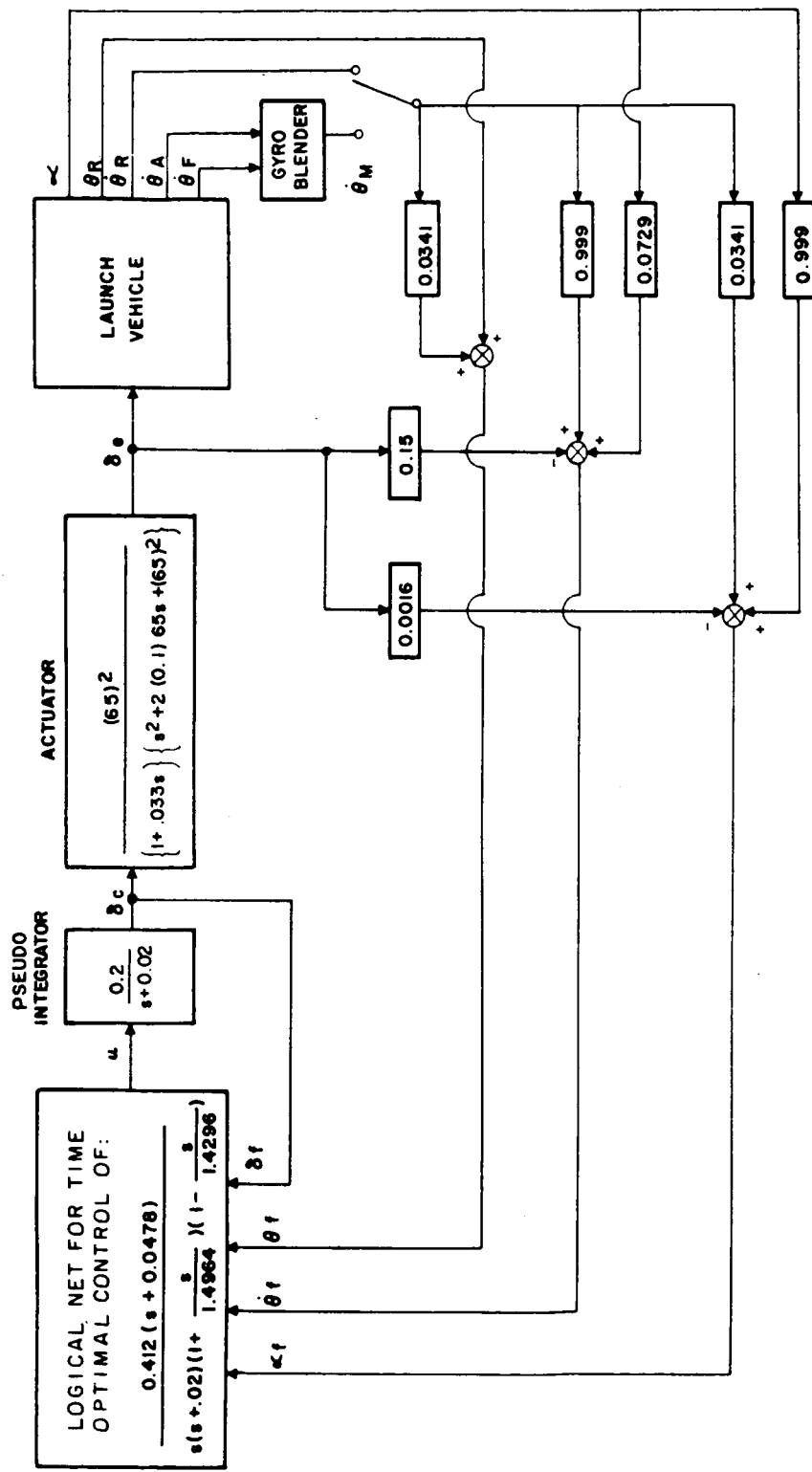


Figure 7. Block Diagram - Fourth-Order Attitude Regulation

the target set. This reduced residual errors due to switching on a quantized switching surface and held the plant within the target set. The linear switching used in this mode was

$$U = \text{sign} \left[\theta_F + 1.25 \dot{\theta}_F + 0.65 \ddot{\theta}_F \right] \quad (9)$$

No attempt was made to minimize the steady state limit cycle with the control variable in this mode.

Two schemes for measurement of the variables fed back to the controller were investigated. In the first the state of the system was measured by the method of Reference 8 in which a complement of n sensors is used in measuring the state of an n^{th} order system. In the second, a rigid-body pitch rate signal was derived by the phase blending technique of Reference 9. This provided a signal which could be freed of first-mode influence; however, in this case a slight amount of first mode feedback was included in the signal to damp the first mode bending.

Typical analog responses are shown in Figures 8, 9, 10, and 11. Rigid-body pitch attitude responses are quite similar for rigid-body feedback and for blender feedback of pitch rate. The small amount of first mode feedback (blender gain $K_1 = 0.9$) causes the first mode to damp out with the blender system whereas the rigid-body feedback there is a sustained oscillation. When the blender gain K_1 was set to cancel all first mode feedback ($K_1 = 1.0$), the blender system also exhibited a sustained oscillation of the first mode. Responses to 40-fps sharp-edged gusts are shown in Figure 10. The single-component attitude regulator essentially ignores the gust disturbance and maintains the desired attitude. Figure 11 illustrates response to various command inputs. Although the system was designed to approximate time-optimal regulation, it exhibits a very good following capability.

CONCLUSION

It has been shown that the collection of experimental procedures and theoretical knowledge is sufficient to use a time-optimal regulation criterion for

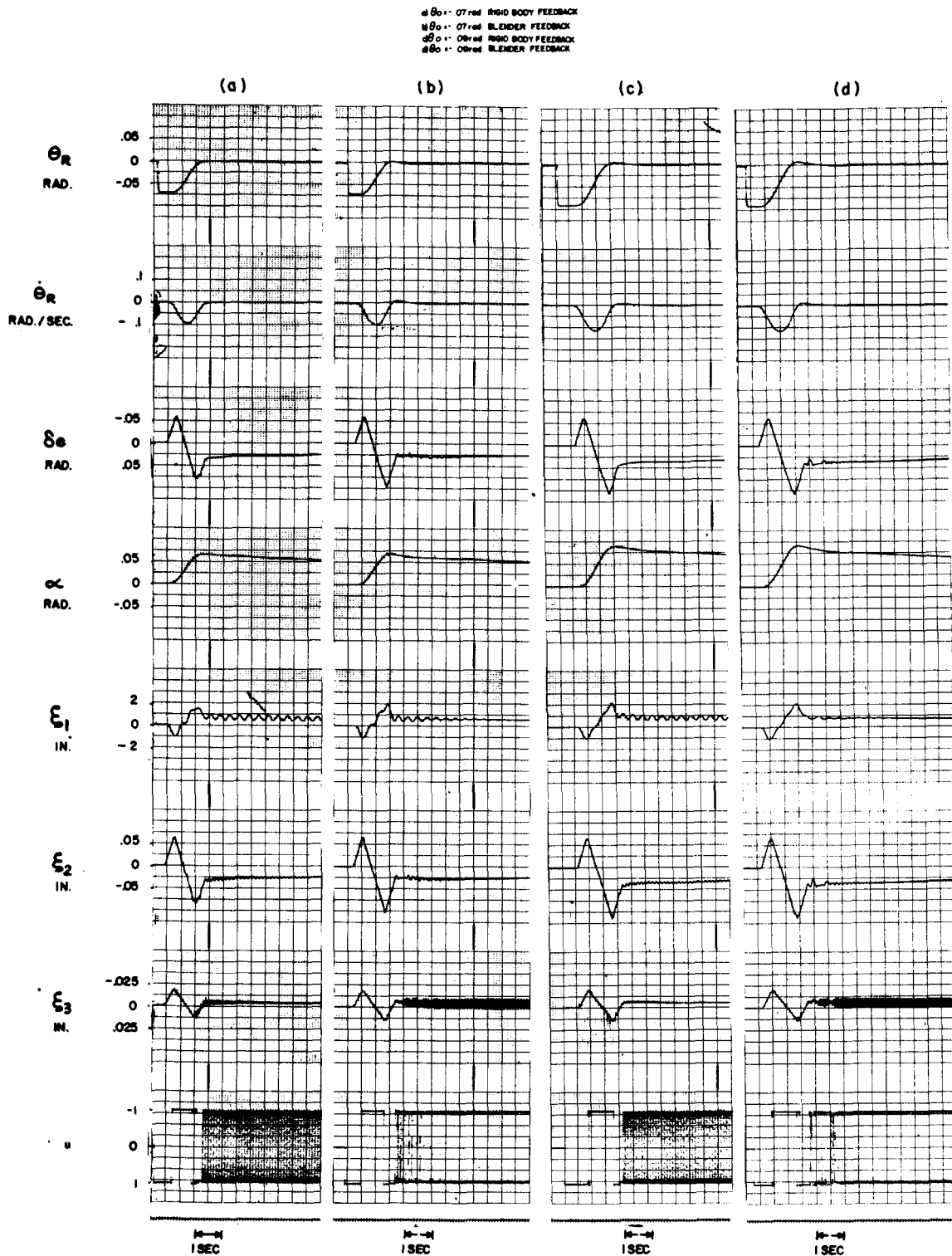


Figure 8. Attitude Regulation - Response to Initial Displacement Errors

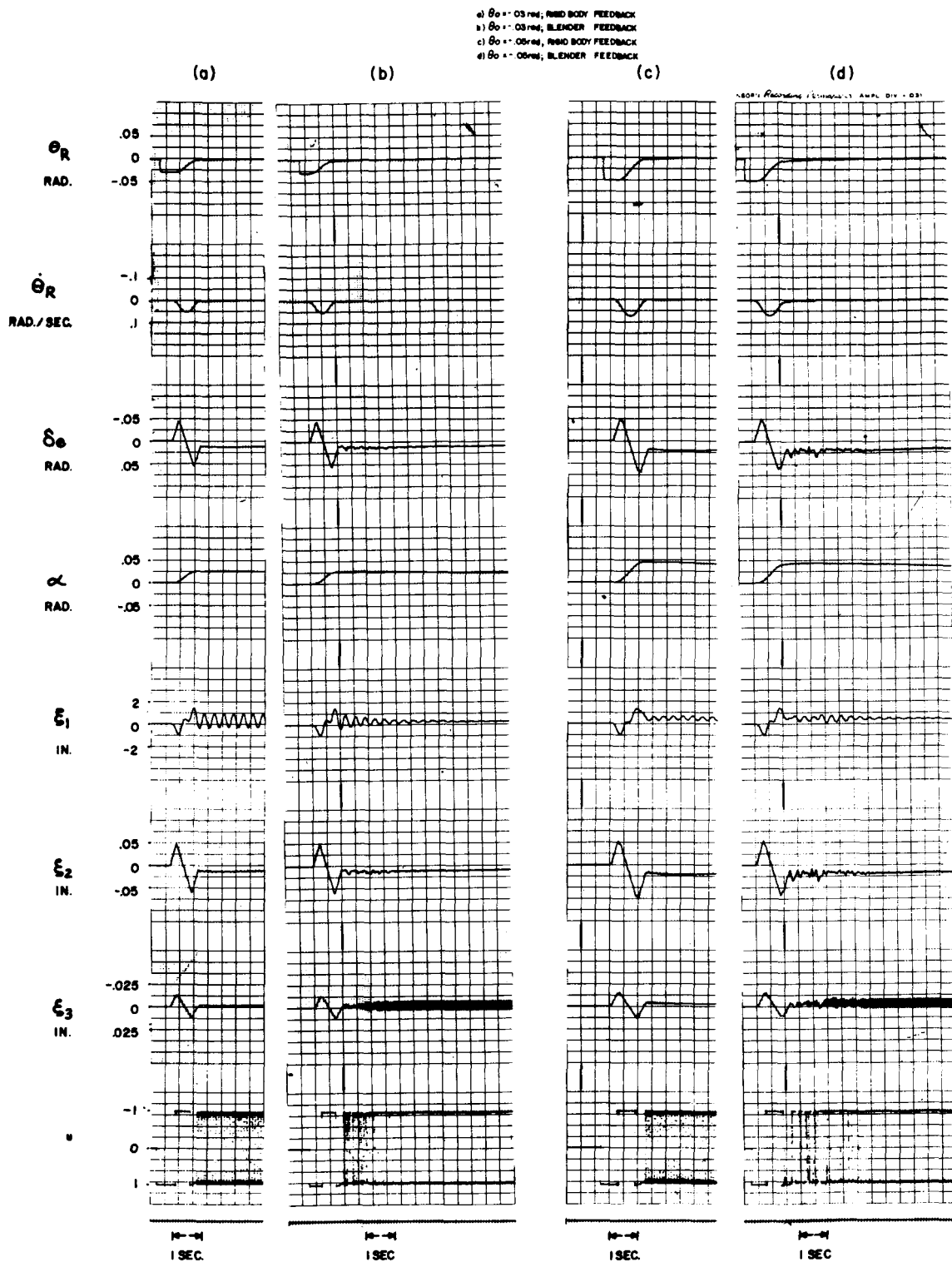


Figure 9. Attitude Regulation - Response to Initial Displacement Errors

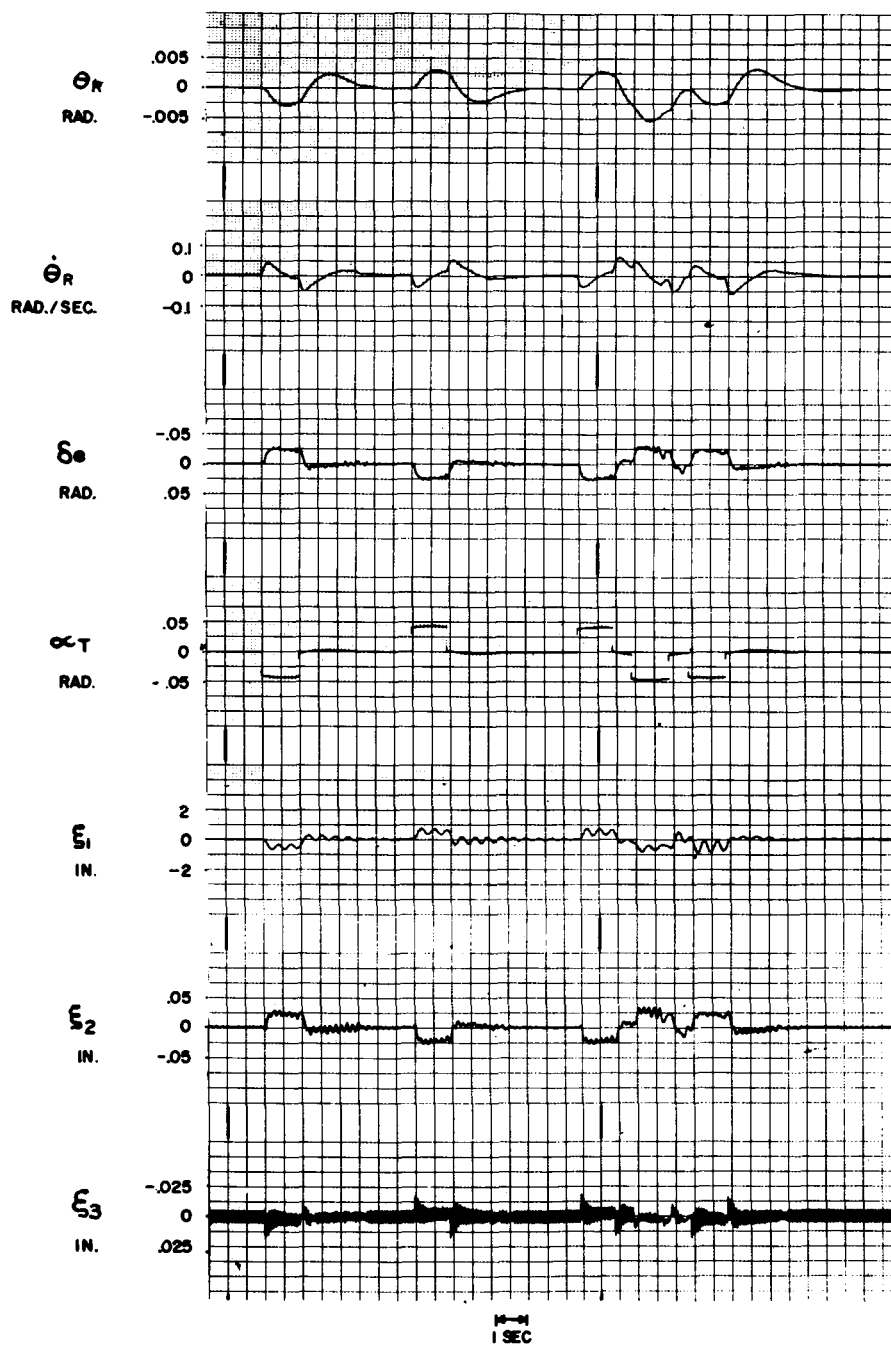


Figure 10. Attitude Regulation - Response to 40 fps Sharp-edge Gust

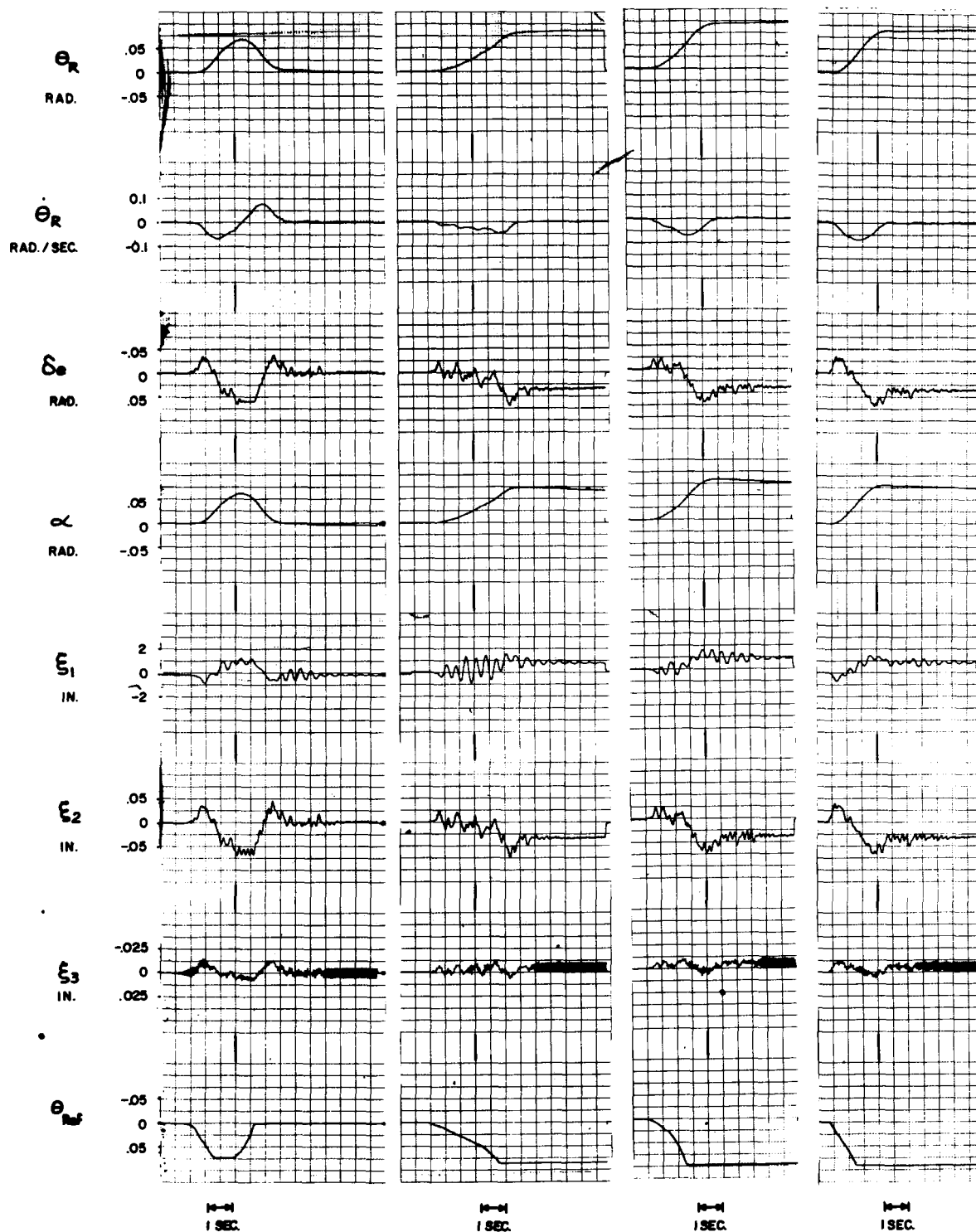


Figure 11. Attitude Control - Response to Command Inputs

rational design of controllers for a high order plant with known coefficients. The synthesis procedure includes obtaining a representative set of open-loop optimum trajectories for a truncated model which is based on the dominant dynamics of the plant. The set of open-loop trajectories is used to define a closed-loop control law for the model. When this controller is applied to the full plant, the output is effectively that of the optimally controlled model filtered by the secondary dynamics of the plant. The resulting controller is relatively simple. However, measurement requirements are severe in that the entire state of the system must be measured. This is feasible with the methods of Reference 8 but undesirable because of the large number of sensors required. Such schemes as the gyroblender give promise of relaxing these requirements.

Minneapolis-Honeywell Regulator Company
Minneapolis, Minnesota
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